Wireless Networks 7, 567–574, 2001

A Graph Theoretic Approach for Channel Assignment in Cellular Networks

MIHAELA IRIDON, DAVID MATULA and CHENG YANG
Computer Science and Engineering Department, Southern Methodist University, Dallas, TX, USA

Abstract. We define a cellular assignment graph to model the channel assignment problem in a cellular network where overlapping cell segments are included in the model. Our main result is the Capacity-Demand Theorem which shows a channel assignment function is always possible unless there is a connected subregion of cells and overlap segments containing more channel requests then the total capacity of all transceivers within or on the boundary of the subregion and covering any part of the subregion with an overlapping segment. We further describe the simplicity and regularity of our proposed cellular assignment graphs and their accessibility for simulation and theoretical investigation without artifacts from the overall geographical region boundaries.

Keywords: cellular assignment graph, overlapping transceiver coverage, triangular lattice model, toroidal embedding

1. Introduction and summary

Channel assignment for wireless mobile units is classically modeled by assuming the coverage regions of transceivers partition the plane into disjoint hexagons [3,7,8,11]. The overlap regions incidental to the coverage regions being more like circles than hexagons are excluded from the fundamental hexagonal lattice. Overlap regions are separately identified with reference to methods of handoff, but the size and variation in overlap regions is not easily investigated in this traditional model [11]. Our approach includes extensive modeling of various forms of overlap segments and their regularities in a cellular arrangement. A new approach to the channel assignment problem in the presence of extensive overlap between coverage regions is obtained by graph theoretic modeling of the cellular assignment problem [6,12,13].

In section 2 we first provide a generic graph model of the assignment problem for arbitrary placement of transceivers and coverage regions over the plane. We then provide a much simplified and regular multipartite graph model for regular cellular transceiver arrangements with edges denoting relations between transceivers and overlap cell segments in their coverage regions. The channel assignment problem on a cellular assignment graph is defined. Our main result is the Cellular Capacity-Demand Theorem and its proof identifying an efficient channel assignment mechanism. The theorem shows that a channel assignment is always possible unless there is a connected region of cells and cell overlap segments with more internal channel assignment requests than the total channels available from all transceivers within and on the boundary of the region and covering any part of the region with an overlap segment.

In section 3, the structure of underlying cellular assignment graphs for various levels of cell overlap are described. A simple regular bipartite graph between triangular overlap regions and boundary transceivers is shown to provide an ideal model for high traffic congested cellular networks with considerable cell overlap.

An embedding of this regular bipartite graph on a torus is given in section 4. This allows that a modest sized finite regular graph is obtained without boundary artifacts for investigating channel assignment strategies both theoretically and by simulation.

In [1] and [9] some surprising channel assignment results are provided for uniform random placement of mobile units into a cellular network employing the cellular assignment results of this paper. The canonical structure and simplicity of our cellular assignment graphs for strong overlap segments suggests that some deeper theoretical probabilistic results may be accessible for associated evolutionary random graph problems [10].

2. Graph models of channel assignment

2.1. Control channel graphs

Let $\mathbf{T} = \{T_j\}$ be a finite set of transceivers distributed so as to cover a geographic region, and let $\mathbf{M} = \{MU_i\}$ be a finite set of mobile units in the region that are in service (turned on for control tracking).

Figure 1 illustrates a geographic region comprising the union of nine coverage regions from nine transceiver sites in the plane. **x**'s denote in-service mobile units. The mobile unit MU_i following the path indicated would be continuously tracked by one or more of transceivers T_8, T_6, T_3, T_5, T_9, T_4 at various times.

The transceiver-mobile *control channel graph* $G(V, E_\tau)$ is a connected bipartite graph with vertices $V = \mathbf{M} \cup \mathbf{T}$ and edges $E_\tau = \{e_{ij} \mid T_j \text{ covers } MU_i \text{ at time } \tau\}$. Thus, $e_{ij} \in E_\tau$ denotes a *control channel* (MU_i, T_j) that may selectively be designated a voice channel in response to a request from MU_i. A mobile may traverse the region from

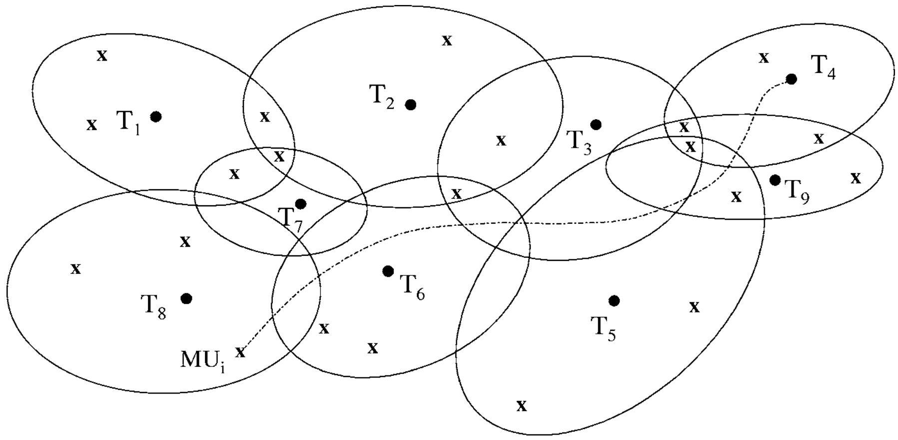

Figure 1. A geographic region with mobile units (**x**'s) covered by from 1 to 4 of the 9 transceivers.

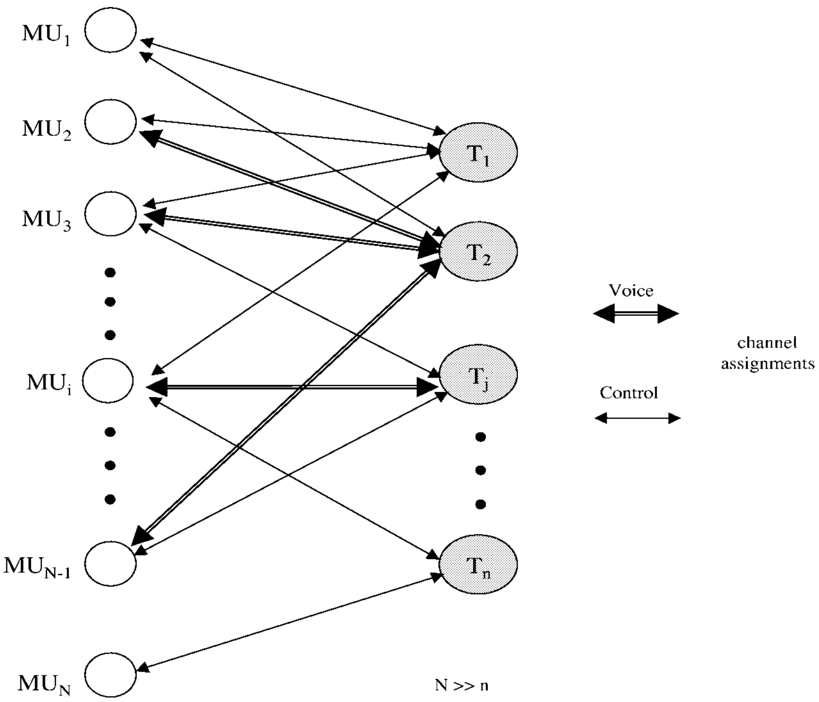

Figure 2. A control channel graph $G(V, E_\tau)$ with four voice channels of E^* highlighted.

any point to any other point and maintain control channels to one or more transceivers for tracking at all times, only if the control channel graph is connected at the time the mobile is about to traverse the region.

Figure 2 illustrates a control channel graph $G(V, E_\tau)$ with $n = |\mathbf{T}|$ transceivers and $N = |\mathbf{M}|$ mobile units in service.

A *star matching* is a subset of edges $E^* \subseteq E(G)$ of a graph G where the induced subgraph $\langle E^* \rangle$ is a forest with every component a star whose center is a transceiver. The edges in a control channel graph $G(V, E, \tau)$ designated as voice channels constitute a star matching. Figure 2 illustrates four of the voice channel edges by highlighted double lines.

The *channel assignment problem* is then: given a control channel graph $G(V, E, \tau)$ where each transceiver has a capacity of k voice channels and where a mobile request subset $MR \subseteq \mathbf{M}$ comprises a set of mobile units requesting a channel assignment at time τ, is there a star matching $E^*(MR)$ comprising $|E^*(MR)| = |MR|$ edges of $G(V, E, \tau)$ with every mobile unit $MU_i \in MR$ incident to one edge of $E^*(MR)$ and every transceiver $T_j \in \mathbf{T}$ incident to at most k edges of $E^*(MR)$?

Theorem 1 (The Channel Assignment Theorem). For the channel assignment problem in $G(V, E, \tau)$ with every transceiver having capacity of k channels, there exists a star matching $E^*(MR)$ for a mobile request set $MR \subseteq \mathbf{M}$ *if and only if* $|M'| \leqslant k|\mathbf{T}(M')|$ for every subset $M' \subseteq MR$, where $\mathbf{T}(M')$ is the set of transceiver vertices adjacent to vertices of M' in $G(V, E, \tau)$.

Proof. The above theorem is a variation of Hall's theorem on systems of distinct representatives [4]. □

The Channel Assignment Theorem is an existential theorem characterizing a solution. Established network flow procedures provide an algorithm for determining the channel assignment that is polynomial time in the size of the graph [2]. The formulation of the channel assignment problem is impractical from the point of view of data structure representation. A typical channel assignment graph may have tens of thousands of vertices corresponding to the in-service mobile units in a metropolitan area. Furthermore, this graph changes dynamically over time presenting difficulties in establishing a data structure to represent the graph. In the next subsection we indicate how the channel assignment problem may be simplified employing the underlying cellular network structure of a mobile communication system. The problem is recast as a cellular channel assignment problem on a fixed graph with variations over time incorporated as changes of weight on selected graph edges.

2.2. Regular cellular networks and cellular assignment graphs

The traditional representation of a cellular communication network is the hexagonal lattice [7,8,11], where each hexagonal cell is covered by a particular transceiver within or on its boundary. Adjacent hexagonal cells are further taken to have an overlap region covered by both corresponding transceivers, where a mobile unit moving between the cells may be handed off in the overlap region to maintain an ongoing call. Figure 3 shows such a hexagonal cellular network with minimal overlap regions explicitly identical.

The regularity of the hexagonal network partitions the plane into a small number of isomorphic types of cell segments identified by the multiple number of transceivers whose coverage areas intersect to define the cell segments. There are just two types of cell segments illustrated in figure 3; the hexagonal star 1-segment denoting a region covered by a single transceiver, and the pointed oval segment denoting an overlap segment covered by exactly two transceivers. Allowing that the ideal coverage area for each transceiver in figure 3 is a circle of minimum radius to cover the plane, about 80% of each hexagon will be in the 1-segment and 20% in a 2-segments in this *minimal overlap* cellular arrangement. Any mobile unit in a particular cell segment has the same set of control channels and is a candidate for the same voice channels. This allows that the same information represented in the unwieldy control channel graph $G(V, E, \tau)$ may be represented by a smaller bipartite graph between transceiver vertices and cell segment vertices, where mobile unit positions and channel assignments are reduced to integer weights on the elements of the graph.

Let $\mathbf{S} = \{s_i\}$ be the set of cell segments defined by overlap regions of a cellular network. A *cellular assignment* graph $G(V, E)$ is a connected bipartite graph with vertices $V = \mathbf{S} \cup \mathbf{T}$ and edges $E = \{e_{ij} \mid$ transceiver T_j covers cell segment $s_i\}$.

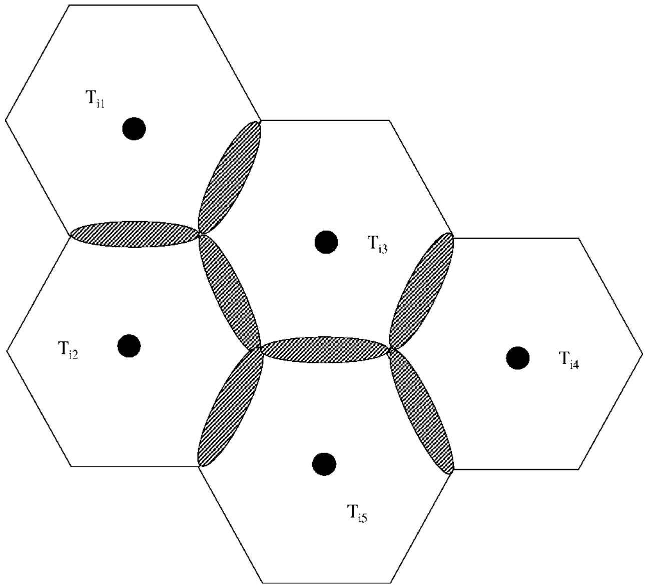

Figure 3. The hexagonal cellular layout with minimal sized overlap regions.

Figure 4 shows a channel assignment graph for the *minimal overlap* cellular arrangement of figure 3. The graph is a regular tripartite graph with three classes of vertices: (i) transceiver vertices of degree 7, (ii) cell 1-segment (hexagonal star) vertices of degree 1, and (iii) cell 2-segments (pointed oval) vertices of degree 2. The regularity derived from the infinite hexagonal lattice on the plane can be preserved in a useful finite version of the graph obtained by wrapping the lattice on a torus as described in section 4. Without an appropriate embedding the boundary of a planar region would introduce irregularities into the graph and likely lead to boundary artifacts in investigations of channel assignments on such graphs.

A *channel assignment* (matching) on a cellular assignment graph $G(V, E)$ is a function $m : E \to \mathbf{Z}$, where $\mathbf{Z}$ is the set of integers, such that $m(e_{ij})$ denotes the number of distinct voice channels of transceiver T_j matched to mobile units in cell segment s_i. Thus, $\sum_i m(e_{ij})$ gives the total number of channels of transceiver T_j currently assigned, and $\sum_j m(e_{ij})$ gives the total number of mobile units residing in cell segment s_i having an assigned voice channel.

The *cellular channel assignment problem* is then: given a channel assignment graph $G(V, E)$ with $V = \mathbf{S} \cup \mathbf{T}$, where each transceiver has a capacity of k voice channels and where there is a demand function $d : \mathbf{S} \to \mathbf{Z}$ requesting channel assignments for $d(s_i)$ distinct mobile units in cell segment $s_i \in \mathbf{S}$ at time τ, is there a channel assignment on $G(V, E)$ satisfying the demand? That is, is there an $m : E \to \mathbf{Z}$ such that $\sum_i m(e_{ij}) \leqslant k$ for all j, where $\sum_j m(e_{ij}) = d(s_i)$ for all i?

An incremental version of channel assignment demand can be used to investigate strategies for channel assignments when choices exist. The *next call* assignment problem assumes that a demand $d_\tau : E \to \mathbf{Z}$ is satisfied by a channel assignment $m_\tau : E \to \mathbf{Z}$ at time τ, and that $d_{\tau+1}(s') = d_\tau(s') + 1$ for some cell segment $s' \in \mathbf{S}$, with

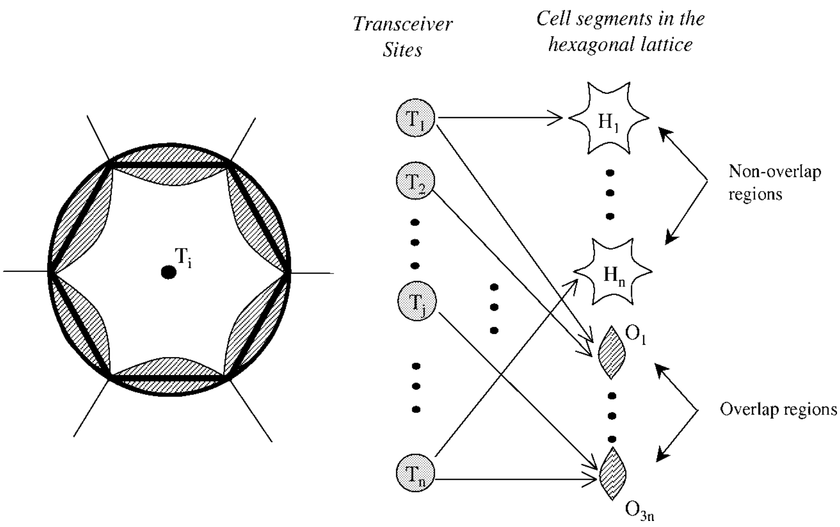

Figure 4. The channel assignment graph model for the hexagonal cellular layout with minimal sized overlap regions.

$d_{\tau+1}(s) = d_\tau(s)$ for $s \in \mathbf{S} - \{s'\}$. So, the demand at time $\tau + 1$ is for one new call to be assigned a channel in cell segment s'. The next call assignment problem asks a sequence of probing questions about cellular assignment as follows:

1. If the demand $d_{\tau+1} : \mathbf{S} \to \mathbf{Z}$ has no solution, what is the nature of the blockage preventing the new call from obtaining a channel assignment?
2. If demand $d_{\tau+1}$ has a channel assignment solution, is there a direct solution assigning an available channel from a transceiver covering s', that is, is there an $m_{\tau+1} : E \to \mathbf{Z}$ such that for some $e' \in E$ incident to $s' \in \mathbf{S} \subset V$, $m_{\tau+1}(e') = 1 + m_\tau(e')$, and $m_{\tau+1}(e) = m_\tau(e), \forall e \in E - \{e'\}$?
3. If the demand $d_{\tau+1}$ has a channel assignment solution but no direct solution, what is the nature of a solution, and is there a "best" solution?

A characterization of the solution of the next call assignment problem is prefaced by several definitions. A *zone* $\langle V' \rangle$ is a connected bipartite subgraph of a cellular assignment graph induced by the vertex set $V' = S' \cup T'$, $S' \subseteq \mathbf{S}$, $T' \subseteq \mathbf{T}$, which is "geometrically closed" in the following sense. If transceiver $T_j \in \mathbf{T}$ is adjacent to (covers) at least one cell segment $s \in S'$, then $T_j \in T'$, and if $s \in \mathbf{S}$ has the adjacent (covering) transceiver T_j belonging to T' for all T_j adjacent to s, then $s \in S'$.

Intuitively, a zone forms a connected geographical region of cell segments including all transceivers internal or close enough to the boundary of the region that its coverage area overlaps the region. The region contains a "hole" only if the hole includes a transceiver $T_j \notin T'$, where none of its covered segments are in V'.

The *capacity*, $cap(V')$, of a zone $\langle V' \rangle$ is the total number of channels $cap(V') = k|T'|$ available for assignment from transceiver vertices of the zone, and the *demand* $d(V') = \sum_{s \in S'} d(s)$ is the total demand over all cell segments of the zone $\langle V' \rangle$.

A necessary condition for a demand $d : \mathbf{S} \to \mathbf{Z}$ to be satisfied by a channel assignment $m : E \to \mathbf{Z}$ in the cellular assignment graph $G(V, E)$ is that $d(V') \leqslant cap(V')$ for every induced subgraph $\langle V' \rangle$ that is a zone of G.

A *blocking* (or *congested*) *zone* $\langle V' \rangle$ for a demand $d : \mathbf{S} \to \mathbf{Z}$ which can be satisfied by a channel assignment is a zone for which the demand equals the capacity, $d(V') = cap(V')$, so that no increase in demand over any cell segment of the zone can be satisfied. The following theorem gives a necessary and sufficient condition for a demand to be satisfied by a channel assignment.

Theorem 2 (The Cellular Capacity-Demand Theorem). The demand $d : \mathbf{S} \to \mathbf{Z}$ is satisfied by a channel assignment $m : E \to \mathbf{Z}$ in a cellular assignment graph $G(V, E)$ *if and only if* $d(V') \leqslant cap(V')$ for every zone $\langle V' \rangle$ of G.

Proof. The proof of the Capacity-Demand theorem follows from a sequential application of the following lemma on next call assignments. □

Lemma 3 (The Next Call Assignment Lemma). Let the demand $d_\tau : \mathbf{S} \to \mathbf{Z}$ be satisfied by the channel assignment $m_\tau : E \to \mathbf{Z}$ for the cellular assignment graph $G(V, E)$ at time τ. Then a next call demand at time $\tau+1$, $d_{\tau+1} : \mathbf{S} \to \mathbf{Z}$, given for a particular $s' \in \mathbf{S}$ by $d_{\tau+1}(s') = 1 + d_\tau(s')$, $d_{\tau+1}(s) = d_\tau(s), \forall s \in \mathbf{S} - \{s'\}$, has a channel assignment $m_{\tau+1} : E \to \mathbf{Z}$ if and only if there is no blocking zone $\langle V' \rangle$ with $s' \in V'$, for the current channel assignment m_τ.

Furthermore, if there is no blocking zone $\langle V' \rangle$ with $s' \in V'$ for $m_\tau : E \to \mathbf{Z}$, then there is an alternating path

$$e_1(s', T_1), e_2(T_1, s_2), e_3(s_2, T_2),$$
$$e_4(T_2, s_3), \ldots, e_{2j+1}(s_{j+1}, T_{j+1})$$

from $s' \in V$ to a transceiver T_{j+1} with an available channel (i.e., $\sum_i m_\tau(e_{i,j+1}) \leqslant k - 1$). Then $m_{\tau+1}(e_{2p+1}) = m_\tau(e_{2p+1}) + 1$ for $p = 0, 1, 2, \ldots, j$, and $m_{\tau+1}(e_{2p}) = m_\tau(e_{2p}) - 1$ for $p = 1, 2, \ldots, j$, and $m_{\tau+1}(e) = m_\tau(e)$ for all $e \in E - \{e_1, e_2, \ldots, e_{2j+1}\}$ provides a channel assignment $m_{\tau+1}$ for $d_{\tau+1}$.

Proof. The proof is based on first establishing a feasible flow in a capacitated network derived from the cellular assignment graph $G(V, E)$. A demand source is joined by an edge to each cell segment $s \in \mathbf{S} \subset V$ and labeled with capacity and flow equal to $d_\tau(s)$. Each edge $e \in E$ is labeled with capacity k and flow $m_\tau(e)$. Each transceiver $T_j \in \mathbf{T} \subset V$ is joined to a channel sink by an edge labeled with capacity k and flow $\sum_i m(e_{ij})$. The flow is then a maximum flow of value $\sum_{s\in\mathbf{S}} d_\tau(s)$ in the capacitated network with the edges from the demand source being a minimum cut. Now increase the capacity of the edge from the demand source s' to $d_\tau(s')+1$, and search for an augmenting path. An augmenting path, if found, provides the alternating path and new channel assignment $m_{\tau+1}$. If not, the reachable set of vertices from the demand source is determined and augmented as required to establish the claimed blocking zone $\langle V'\rangle$. □

The search for an augmenting path described in the proof sketch can be performed by a breadth first search in the graph G respecting the edge weights m_τ. This yields a minimum length path $e_1, e_2, \ldots, e_{2j+1}$ satisfying the conditions of the lemma when an augmenting path exists. Such a path is termed a *handoff chain* as j handoffs are utilized in establishing the new channel assignment $m_{\tau+1}$. For $j = 0$, a direct channel assignment is possible. When no augmenting path exists, the breadth first search finds a reached set of vertices readily augmented to define the blocking zone $\langle V'\rangle$.

Observation 4. Given a channel assignment $m_\tau : E \rightarrow \mathbf{Z}$ for the demand $d_\tau : \mathbf{S} \rightarrow \mathbf{Z}$ in the cellular assignment graph $G(V, E)$, and a next call cell segment s' where $d_{\tau+1}(s') = d_\tau(s') + 1$, a breadth first search of a portion of the graph G finds either:

(i) a blocking (congested) zone $\langle V'\rangle$ with $s' \in V'$ and does so in time $O(|V'|)$, since the degree of the graph is fixed and is equal to 6, or

(ii) a satisfying channel assignment $m_{\tau+1}$ employing at most j handoffs and does so in time $O(j^2)$. This results from the fact the at depth d the BFS algorithm would visit $6d$ new vertices and hence the total search time for finding a chain of at most j handoffs is proportional to j^2.

3. Overlap levels in regular cellular networks

Defining unit distance in a traditional hexagonal cellular network as the distance between hexagonal cell centers, the overlap regions of figure 3 are determined by circular transceiver coverage regions of radius $r = \sqrt{3}/3$ sufficient to reach the hexagonal corners of the cells.

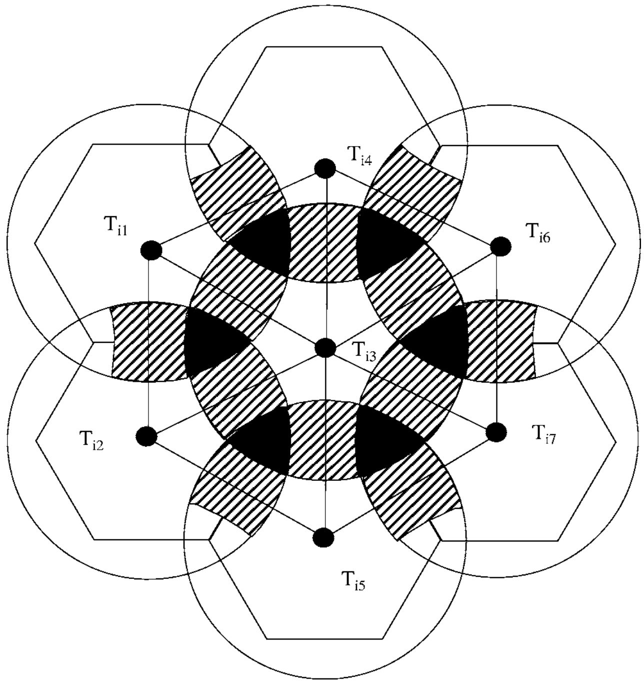

Figure 5. The dual graph of the hexagonal cellular layout with stronger overlap regions.

In practice, to provide handoff regions of sufficient extent for mobile units in the neighborhood of corners of the hexagons, coverage regions some 15 to 20% greater are needed. Figure 5 employs such circular coverage regions of radius $r = 0.7$ ($\simeq$20% larger radius) and yields representative cell segments of three types:

(i) 1-segments are hexagonal stars and cover some 40% of the region;

(ii) 2-segments are "rectangles" formed by two convex arcs and two concave arcs and cover some 40% of the region;

(iii) 3-segments are "triangles" formed by three convex arcs and cover some 20% of the region.

The cellular overlap arrangement illustrated in figure 5 is effectively a weak overlap arrangement.

In mature cellular systems for metropolitan areas with many relatively close transceivers, it is common to have coverage regions exhibiting greater overlap. For example, the cell segments of figure 6(b) correspond to radius $r = 0.9$, where some 80% of the region is in a nearly triangular cell segment with the rest of the region distributed over small 1-, 2-, and 4-segments. Figure 6 employs a representative triangle of the planar dual triangular grid (also shown in figure 5) to illustrate the proportional i-segment sizes for weak overlap, $r = 0.64$ in figure 6(a), and strong overlap, $r = 0.90$ in figure 6(b).

Figure 7 shows the portion of service coverage in i-segments for $r = 0.1$ to 1.1, indicating the peaks for mostly single coverage in weak overlap arrangements and mostly triple coverage in strong overlap arrangements. We note

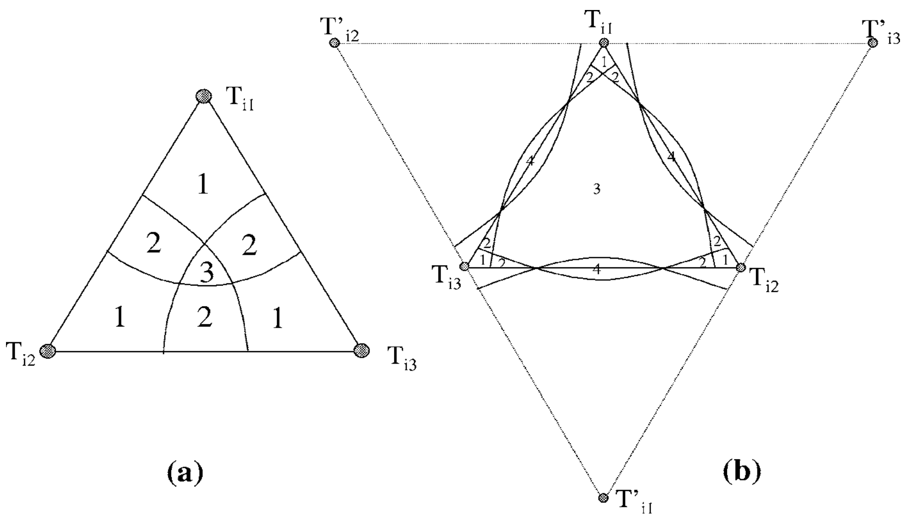

Figure 6. Transceiver coverage in a dual triangular cell. (a) $r \simeq 0.64$; (b) $r \simeq 0.90$.

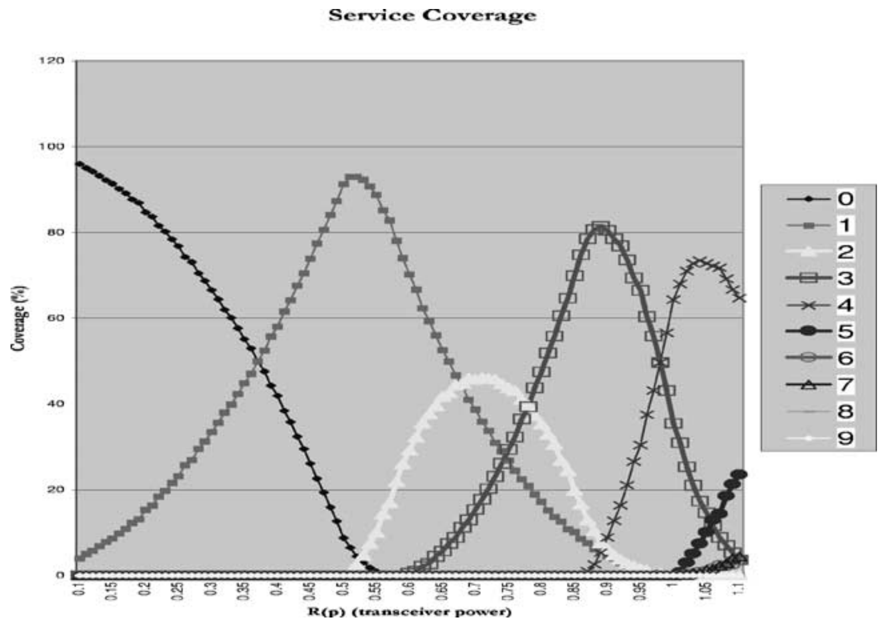

Figure 7. Transceiver coverage distribution in terms of cell radius.

that a similar overlap arrangement result can be obtained for regions formed by 120° directional antenna broadcasts as well as by 360° omni-directional broadcasts from transceiver sites. Note that for any radius $\sqrt{3}/3 \leqslant r \leqslant 1$, the cell segments are of only four symmetric types, designated as 1-segments, 2-segments, 3-segments and 4-segments. The corresponding bipartite cellular assignment graph is then always a regular 5-partite graph, having at most five classes of vertices with each class having vertices all of the same degree.

The strong overlap arrangement of figure 6(b) suggests a good approximation is simply to have the full triangles of the planar dual triangular lattice serve as the overlap 3-segments, with no other cell segments admitted. This yields the ideal cellular assignment graph of figure 8 which is regular bipartite with each triangle vertex adjacent to the three transceiver vertices at the corners of the planar dual triangular lattice, and each transceiver vertex incident to six triangle vertices in the cellular assignment graph. It is interesting to notice here that the geometric regularity of the coverage/overlap model leads to the regularity of the cellular assignment graph model. To avoid boundary anomalies destroying this regularity in a representative finite graph, we shall wrap the triangular lattice on the torus by a symmetric embedding.

4. Toroidal embedding and finite regular cellular assignment graphs

The embedding of the infinite planar hexagonal lattice on the torus is best illustrated by employing the planar dual triangular lattice. Rather than employing a Cartesian rectangu-

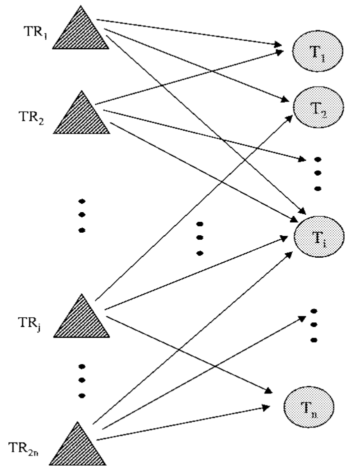

Figure 8. The bipartite cellular assignment graph of the triangular layout.

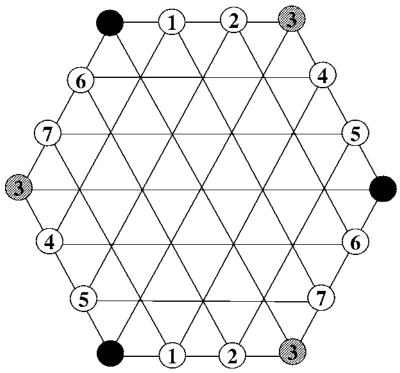

Figure 9. A hexagonal repeat region of the toroidal embedding.

lar repeat wrapped top to bottom and left to right, greater symmetry is obtained by having the repeat pattern itself be a hexagon of triangular lattice cells, as illustrated in figure 9. The wrapping identifies opposite sides of the hexagonal repeat pattern as shown and is obtainable by cut and paste methods as illustrated in [5]. Such a hexagonal repeat pattern is identified by the distance r from the center to the boundary, with $r = 3$ in figure 9. There are then $6r^2$ triangular cells in the repeat pattern, and $3r^2$ distinct triangular grid vertices, noticing the double and triple repeat grid vertices shown on the repeat boundary in figure 9. The ideal strong overlap Cellular Assignment graph $G_r(V, E)$ corresponding to the hexagonal repeat pattern of the embedding is a regular bipartite toroidal graph with the $3r^2$ triangular grid vertices as transceiver vertices **T**, and the $6r^2$ triangular cells as cell segment vertices **S**, with $V = \mathbf{S} \cup \mathbf{T}$.

Observation 5. The toroidal Cellular Assignment graph $G_r(V, E)$ is a regular bipartite graph on $V = \mathbf{S} \cup \mathbf{T}$ with the $|T| = 3r^2$ vertices of **T** having degree six and the $|S| = 6r^2$ vertices of **S** having degree three. There are $9r^2$ edges in E.

Observation 6. The toroidal Cellular Assignment graph $G_r(V, E)$ is edge symmetric. $G_r(V, E)$ is partially vertex symmetric in that there is an isomorphism of $G_r(V, E)$ mapping any vertex of $\mathbf{T} \subset V$ into any other vertex of **T**, and an isomorphism mapping any vertex of $\mathbf{S} \subset V$ into any other vertex of **S**.

The existence of this extensive symmetry provides a simple canonical graph for investigation both theoretically and by simulation. Implementation of channel assignments by distributed algorithms replicated at the transceivers and in the mobile units is a beneficial by-product of this symmetry.

References

[1] H.C. Cankaya, M. Iridon and D.W. Matula, Performance analysis of a graph model for channel assignment in a cellular network, in: *COMPSAC* (1999).

[2] L.R. Ford, Jr., and D.R. Fulkerson, *Flows in Networks* (Princeton University Press, 1962).

[3] J. Hale, Frequency assignment, IEEE Proceedings 68(12) (December 1980).

[4] P. Hall, On representatives of subsets, Journal of the London Mathematical Society 10 (1935) 26–30.

[5] M. Iridon and D.W. Matula, Simulating cellular system behavior without boundary effects by embedding the network model on a torus, in: *Proceedings of the ICCCN 1998*, Lafayette, LA (1998).

[6] M. Iridon, Regular triangulated toroidal graphs with applications in cellular and interconnection networks, Ph.D. Thesis, Southern Methodist University (1999).

[7] I. Katzela and M. Naghshineh, Channel assignment schemes for cellular mobile telecommunication systems: A comprehensive survey, IEEE Personal Communications (June 1996).

[8] V.H. MacDonald, AMPS: The cellular concept, The Bell System Technical Journal 58(1) (January 1979) 5–41.

[9] D.W. Matula, M. Iridon, C. Yang and H.C. Cankaya, A graph theoretic approach for channel assignment in cellular networks, in: *2nd International Workshop on Discrete Algorithms and Methods for Mobile Computing and Communications* (1998). In conjunction with *ACM/IEEE Mobicom 1998*.

[10] E.M. Palmer, *Graphical Evolution, An Introduction to the Theory of Random Graphs*, Wiley-Interscience Series in Discrete Mathematics (1985).

[11] G.P. Pollini, Trends in handover design, IEEE Communications Magazine (March 1996).

[12] C. Yang, A multi-layer design and load sharing algorithm for personal communication networks, Ph.D. Thesis, Southern Methodist University (1991).

[13] C. Yang and D.W. Matula, Multi-layered arrangement for load sharing in a cellular communication system, U.S. Patent #5,633,915 (May 1997).

Mihaela Iridon received her B.S. in mathematics and physics from the Gh. Lazar College and the M.S. in computer engineering from the L. Blaga University, both in Sibiu, Romania. She also received a M.S. and a Ph.D. in computer science from Southern Methodist University in Dallas in 1997 and 1999, respectively. Her thesis topic was in graph theory with applications in cellular and interconnection networks. Dr. Iridon has taught for two years at the L. Blaga University, Computer Engineering Department, in Sibiu, Romania (1994–1995) and has worked as a teaching assistant for 3.5 years at Southern Methodist University (1996–1999). She is currently employed as a software engineer at Verizon, Call Center Services in Irving, Texas, doing development and research in computer telephony integration areas, while continuing the research in graph theory and cellular networks.

David W. Matula received his B.S. from Washington University, St. Louis, in 1959 and the Ph.D. from U.C. Berkeley, in 1966. He has been a Professor of Computer Science and Engineering at Southern Methodist University in Dallas since 1974. He has held visiting positions at IBM T.J. Watson Research Center, Stanford University, the Naval Postgraduate School, and the University of Texas in the US, and overseas at universities in Karlsruhe, Aarhus, Frankfurt, Lyon, and Odense. He has published some 100 journal and proceedings papers in the areas of computer arithmetic, algorithms design, graph algorithms, random structures, and cluster analysis, and holds a dozen patents. Dr. Matula served on the founding editorial boards of the Journal of Classification, the ORSA Journal on Computing, and the journal Random Structures and Algorithms. He has served on numerous IEEE conference program committees and was a General Chairman for the 15th IEEE Symposium on Computer Arithmetic in 2001.

C. Yang. Photograph and biography not available at time of publication.